Doubly heavy systems: decays and OPE.

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Abstract

We discuss questions related to the application of OPE for the long-lived systems with two heavy flavors. The values of quark masses entering such calulations are constrained by use of currently available data on the lifetimes of B_d and B_c-mesons. The lifetimes of doubly charmed baryons at these values of parameters are evaluated. The necessary comments on the difference in the results obtained previously are made.

1   Introduction

The main motivation for studying the weak decays of doubly heavy systems is to test our understanding of QCD in the limit, where some masses involved are heavy and, as a consequence, certain aspects of bound state dynamics are simplified. In the case, when the decay of bound system proceeds due to electro-weak interactions, the consideration also gives us a possibility to extract some basic properties of quark interactions at a fundamental level, including precise determination of CKM parameters. The analysis of decays for the doubly heavy systems already has a long history. First of all, the decays of J/ψ and Υ were considered. Being composed by the quark and antiquark of the same flavor, these mesons decay mainly trough quark-antiquark annihilation into hadrons or lepton pair. Another particle, acquiring a lot of attention, was the B_c-meson [2]. It represents a first long-lived particle in the family of doubly heavy systems, whose decays proceed due to the weak interactions. The other representatives in the family of doubly heavy hadrons are baryons with two heavy quark, yet to be discovered experimentally. In this paper we would like to consider lifetimes and some issues of OPE, used in such the framework for the hadrons with two heavy quarks, which decay due to the weak interactions.

Weak decays of the ground state of B_c-meson together with semileptonic and various exclusive modes were considered in [3, 4, 5, 7, 8, 9]. The estimates of total B_c-lifetime in various quark models were done in [4, 5, 6]. The first OPE based result for the lifetime of B_c-meson was obtained in [10], where, however, the matrix element for the dimension 5 operator $\bar{Q}g\sigma \cdot GQ$ was incorrectly evaluated, which led to the overestimation of B_c-lifetime[1]. In addition, the leading corrections to the spectator decays of $\bar{b}$ and c-quarks determined by the kinetic energy, weak annihilation and Pauli interference, were not calculated explicitly in [10]. A systematical OPE-based approach to the evaluation of B_c-meson lifetime was developed in [11]. In a short time, the lifetime value predicted in

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[1]See a discussion of this point in [11].

[11] was experimentally confirmed by the discovery of B_c-meson at FNAL by the CDF-collaboration [1].

At the next stage in studying the decays of doubly heavy systems the approach of [11] was generalized to the case of baryons with two heavy quarks in [12, 13]. A repetition of our results at different values of parameters was done in [14]. The questions related to the spectroscopy, sum rules and production of doubly heavy baryons were considered in [15, 16, 17, 18], correspondingly.

In this work we discuss the question related to the choice of parameters used in the OPE-based frameworks for the doubly heavy systems, as required to obtain the theoretical predictions on the lifetimes in agreement with the experimentally measured values.

This paper is organized as follows. In Section 2, we define notations in the framework of OPE-calculations for the total widths of hadrons with two heavy quarks. In Section 3, the procedure for estimatinf the non-perturbative matrix elements between the states with two heavy flavors is considered. Section 4 is devoted to the numerical evaluation and discussion of parameter choice explored for the calculation of lifetimes for the doubly charmed baryons to be consistent with the experimentally measured value of B_c-meson lifetime. We draw conclusions in Section 5 by summarizing our results.

2 The OPE framework for lifetimes

In accordance with the optical theorem, the total width Γ_H for the hadron H, where H is the B_c-meson or one of the baryons Ξ_{cc}^{++}, Ξ_{cc}^{+} and Ω_{cc} , has the form

$$\Gamma_H = \frac{1}{2M_H}\langle H|\mathcal{T}|H\rangle, \tag{1}$$

where we accept the ordinary relativistic normalization of state, $\langle H|H\rangle = 2EV$, and the transition operator $\mathcal{T}$:

$$\mathcal{T} = \Im m \int d^4x \, \{\hat{T}H_{eff}(x)H_{eff}(0)\}, \tag{2}$$

is determined by the effective lagrangian of weak interaction H_{eff} at the characteristic hadron energies:

$$H_{eff} = \frac{G_F}{2\sqrt{2}}V_{q_2q_3}V_{Qq_1}^*[C_+(\mu)O_+ + C_-(\mu)O_-] + h.c. \tag{3}$$

where

$$O_\pm = [\bar{q}_{1\alpha}\gamma_\nu(1-\gamma_5)Q_\beta][\bar{q}_{2\gamma}\gamma^\nu(1-\gamma_5)q_{3\delta}](\delta_{\alpha\beta}\delta_{\gamma\delta} \pm \delta_{\alpha\delta}\delta_{\gamma\beta}),$$

$$C_+ = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{\frac{6}{33-2f}}, \quad C_- = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{\frac{-12}{33-2f}},$$

so that f denotes the number of flavors, and Q marks the flavor of heavy quark (b or c).

The quantity $\mathcal{T}$ in (1) permits the Operator Product Expansion in the inverse powers of heavy quark mass. The reason is that, the energy release in the weak decay of either quark is large compared to the scale of bound state dynamics, and, so, we can expand in series over the ratio of these scales. In this way, the OPE has the form:

$$\mathcal{T} = \sum_{i=1}^{2}\{C_1(\mu)\bar{Q}^iQ^i + \frac{1}{m_{Q^i}^2}C_2(\mu)\bar{Q}^ig\sigma_{\mu\nu}G^{\mu\nu}Q^i + \frac{1}{m_{Q^i}^3}O(1)\}. \tag{4}$$

The leading contribution is given by the spectator decay, i.e. by the term $\bar{Q}Q$, which is the operator of dimension 3. The corrections to the spectator decays of quarks are given by the operator of dimension 5: $Q_{GQ} = \bar{Q}g\sigma_{\mu\nu}G^{\mu\nu}Q$ and operators of dimension 6, $Q_{2Q2q} = \bar{Q}\Gamma q\bar{q}\Gamma^{'}Q$, where the dominant contributions are provided by the Pauli interference and weak scattering. For the latter, we have

$$
\begin{aligned}
\mathcal{T}_{B_c^+} &= \mathcal{T}_{35b} + \mathcal{T}_{35c} + \mathcal{T}_{6,PI}^{(1)} + \mathcal{T}_{6,WA}^{(1)}, \\
\mathcal{T}_{\Xi_{cc}^{++}} &= 2\mathcal{T}_{35c} + \mathcal{T}_{6,PI}^{(2)}, \\
\mathcal{T}_{\Xi_{cc}^{+}} &= 2\mathcal{T}_{35c} + \mathcal{T}_{6,WS}^{(3)}, \\
\mathcal{T}_{\Omega_{cc}^{+}} &= 2\mathcal{T}_{35c} + \mathcal{T}_{6,PI}^{(4)},
\end{aligned}
$$

where $\mathcal{T}_{35Q}$ denotes the contributions into the decays of quark Q by the operators with the dimensions 3 and 5, and the forthcoming terms are the interference, scattering and annihilation of constituents. In the explicit form we find

$$
\mathcal{T}_{35b} = \Gamma_{b,spec}\bar{b}b - \frac{\Gamma_{0b}}{m_b^2}[2P_{c1} + P_{c\tau1} + K_{0b}(P_{c1} + P_{cc1}) + K_{2b}(P_{c2} + P_{cc2})]O_{Gb}, \tag{5}
$$

$$
\mathcal{T}_{35c} = \Gamma_{c,spec}\bar{c}c - \frac{\Gamma_{0c}}{m_c^2}[(2 + K_{0c})P_{s1} + K_{2c}P_{s2}]O_{Gc}, \tag{6}
$$

where

$$
\Gamma_{0b} = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2, \qquad \Gamma_{0c} = \frac{G_F^2 m_c^5}{192\pi^3} \tag{7}
$$

with $K_{0Q} = C_-^2 + 2C_+^2$, $K_{2Q} = 2(C_+^2 - C_-^2)$, and $\Gamma_{Q,spec}$ denotes the spectator width (see [19, 21, 22, 23]):

$$
P_{c1} = (1 - y)^4, \quad P_{c2} = (1 - y)^3, \tag{8}
$$

$$
\begin{aligned}
P_{c\tau1} &= \sqrt{1 - 2(r + y) + (r - y)^2}[1 - 3(r + y) + 3(r^2 + y^2) - r^3 - y^3 - 4ry + \\
&\quad 7ry(r + y)] + 12r^2y^2 \ln\frac{(1 - r - y + \sqrt{1 - 2(r + y) + (r - y)^2})^2}{4ry}
\end{aligned} \tag{9}
$$

$$
P_{cc1} = \sqrt{1 - 4y}(1 - 6y + 2y^2 + 12y^3)24y^4 \ln\frac{1 + \sqrt{1 - 4y}}{1 - \sqrt{1 - 4y}} \tag{10}
$$

$$
P_{cc2} = \sqrt{1 - 4y}(1 + \frac{y}{2} + 3y^2) - 3y(1 - 2y^2) \ln\frac{1 + \sqrt{1 - 4y}}{1 - \sqrt{1 - 4y}} \tag{11}
$$

where $y = \frac{m_c^2}{m_b^2}$ and $r = m_\tau^2/m_b^2$. The functions $P_{s1}(P_{s2})$ can be obtained from $P_{c1}(P_{c2})$ by the substitution $y = m_s^2/m_c^2$. In the b-quark decays, we neglect the value m_s^2/m_b^2 and suppose $m_s = 0$.

The calculation of Pauli interference for the products of heavy quark decays with the quarks in the initial state, weak scattering and annihilation of quarks, composing the hadron, results in:

$$
\begin{aligned}
\mathcal{T}_{6,PI}^{(1)} &= \mathcal{T}_{PI,c\bar{s}}^{\bar{b}} \tag{12} \\
\mathcal{T}_{6,WA}^{(1)} &= \mathcal{T}_{WA,c\bar{s}} + \mathcal{T}_{WA,u\bar{d}} + \sum_l \mathcal{T}_{WA,\nu_l\bar{l}} \tag{13}
\end{aligned}
$$

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$$\mathcal{T}_{6,PI}^{(2)} = 2\mathcal{T}_{PI,u\bar{d}}^{c} \tag{14}$$

$$\mathcal{T}_{6,WS}^{(3)} = 2\mathcal{T}_{WS,cd} \tag{15}$$

$$\mathcal{T}_{6,PI}^{(4)} = 2\mathcal{T}_{PI,u\bar{d}}^{c'} + 2\sum_{l}\mathcal{T}_{PI,\nu_l\bar{l}}^{c} \tag{16}$$

so that

$$2\mathcal{T}_{PI,u\bar{d}}^{c} = -\frac{G_F^2|\Psi^{dl}(0)|^2}{4\pi}m_c^2(1-\frac{m_u}{m_c})^2(m_c+m_u)\left\{10(1-z_-)^2 - \frac{17}{3}(1-z_-)^2\right\} \times$$
$$\left((C_+ + C_-)^2 + \frac{1}{3}(1-4k^{\frac{1}{2}})(5C_+^2 + C_-^2 - 6C_-C_+)\right) \tag{17}$$

$$2\mathcal{T}_{WS,cd} = \frac{3G_F^2|\Psi^{dl}(0)|^2}{\pi}m_c^2(1+\frac{m_d}{m_c})^2(m_c+m_d)(1-z_+)^2 \times$$
$$\left(C_+^2 + C_-^2 + \frac{1}{3}(1-4k^{\frac{1}{2}})(C_+^2 - C_-^2)\right) \tag{18}$$

$$2\mathcal{T}_{PI,u\bar{d}}^{c'} = \frac{13G_F^2|\Psi^{dl}(0)|^2}{12\pi}m_c^2(1-\frac{m_s}{m_c})^2(m_c+m_s) \times$$
$$\left((C_+ - C_-)^2 + \frac{1}{3}(1-4k^{\frac{1}{2}})(5C_+^2 + C_-^2 + 6C_-C_+)\right) \tag{19}$$

$$2\mathcal{T}_{PI,\nu_\tau\bar{\tau}}^{c} = \frac{G_F^2|\Psi^{dl}(0)|^2}{\pi}m_c^2(1-\frac{m_s}{m_c})^2(m_c+m_s)\left\{10(1-z_-)^2 - \frac{17}{3}(1-z_-)^2\right\} \tag{20}$$

$$\mathcal{T}_{PI,c\bar{s}}^{\bar{b}} = \frac{G_F^2}{12\pi}|V_{cb}|^2f_{B_c}^2 M_{B_c}(m_b - m_c)^2(1-z_-)^2(2C_+^2 - C_-^2) \tag{21}$$

$$\mathcal{T}_{WA,cs} = \frac{G_F^2}{24\pi}|V_{cb}|^2f_{B_c}^2 M_{B_c}m_c^2(1-z_+)^2(4C_+^2 + C_-^2 + 4C_+C_-) \tag{22}$$

$$\mathcal{T}_{WA,\nu\tau} = \frac{G_F^2}{8\pi}|V_{cb}|^2f_{B_c}^2 M_{B_c}m_\tau^2(1-z_\tau)^2 \tag{23}$$

$$\mathcal{T}_{PI,\nu_e\bar{e}}^{c} = \mathcal{T}_{PI,\nu_\mu\bar{\mu}}^{c} = \mathcal{T}_{PI,\nu_\tau\bar{\tau}}^{c}(z_\tau \to 0) \tag{24}$$

Here $\Psi(0)$ is the value of quark-diquark baryon wavefunction at the origin, and f_{B_c} is the leptonic constant for B_c-meson. In the evolution of coefficients C_+ and C_-, we have taken into account the threshold effects, connected to the heavy quark masses.

In expressions (5) and (6), the scale μ has been taken approximately equal to m_c. In the Pauli interference term, we suggest that the scale can be determined on the basis of the agreement of the experimentally known difference between the lifetimes of Λ_c, Ξ_c^+ and Ξ_c^0 with the theoretical predictions in the framework described above[2]. In any case, the choice of the normalization scale leads to uncertainties in the final results.

At present, the spectator decays of heavy quarks, contributing to $\mathcal{T}_{35Q}$, are known in the logarithmic approximation of QCD to the second order [24, 25, 26, 27, 28], including the mass corrections in the final state with the charmed quark and τ-lepton [28] in the decays of b-quark and with the strange quark mass for the decays of c-quark. In the numerical estimates, we include these corrections and mass effects, but we neglect the decay modes suppressed by the Cabibbo angle, and also the strange quark mass effects in the b-decays.

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[2]A more expanded description is presented in [12].

Thus, the calculation of lifetimes for the doubly heavy systems under consideraton is reduced to the problem of evaluating the matrix elements of operators, which is the subject of next section.

3   Hadronic matrix elements

By use of motion equations, the matrix element of operator $\bar{Q}^j Q^j$ can be expanded in series over powers of $1/m_{Q^j}$:

$$\langle H|\bar{Q}^j Q^j|H\rangle_{norm} = 1 - \frac{\langle H|\bar{Q}^j[(i\boldsymbol{D})^2 - (\frac{i}{2}\sigma G)]Q^j|H\rangle_{norm}}{2m_{Q^j}^2} + O(\frac{1}{m_{Q^j}^3}). \tag{25}$$

Thus, we have to estimate the matrix elements of operators from the following list only:

$$\bar{Q}^j(i\boldsymbol{D})^2 Q^j, \quad (\frac{i}{2})\bar{Q}^j \sigma G Q^j, \quad \bar{Q}^j \gamma_\alpha(1 - \gamma_5)Q^j \bar{q}\gamma^\alpha(1 - \gamma_5)q,$$
$$\bar{Q}^j \gamma_\alpha \gamma_5 Q^j \bar{q}\gamma^\alpha(1 - \gamma_5)q, \quad \bar{Q}^j \gamma_\alpha \gamma_5 Q^j \bar{Q}^k \gamma^\alpha(1 - \gamma_5)Q^k, \tag{26}$$
$$\bar{Q}^j \gamma_\alpha(1 - \gamma_5)Q^j \bar{Q}^k \gamma^\alpha(1 - \gamma_5)Q^k.$$

The meaning of each term in the above list was already discussed in the previous papers on the decays of doubly heavy baryons [12, 13] and in [11], so we omit it here.

Further, employing the NRQCD expansion of operators $\bar{Q}Q$ and $\bar{Q}g\sigma_{\mu\nu}G^{\mu\nu}Q$, we have

$$\bar{Q}Q = \Psi_Q^\dagger \Psi_Q - \frac{1}{2m_Q^2}\Psi_Q^\dagger(i\boldsymbol{D})^2\Psi_Q + \frac{3}{8m_Q^4}\Psi_Q^\dagger(i\boldsymbol{D})^4\Psi_Q -$$
$$\frac{1}{2m_Q^2}\Psi_Q^\dagger g\boldsymbol{\sigma B}\Psi_Q - \frac{1}{4m_Q^3}\Psi_Q^\dagger(\boldsymbol{D}g\boldsymbol{E})\Psi_Q + ... \tag{27}$$

$$\bar{Q}g\sigma_{\mu\nu}G^{\mu\nu}Q = -2\Psi_Q^\dagger g\boldsymbol{\sigma B}\Psi_Q - \frac{1}{m_Q}\Psi_Q^\dagger(\boldsymbol{D}g\boldsymbol{E})\Psi_Q + ... \tag{28}$$

Here the factorization at scale μ ($m_Q > \mu > m_Q v_Q$) is supposed. We have omitted the term of $\Psi_Q^\dagger \boldsymbol{\sigma}(g\boldsymbol{E}\times\boldsymbol{D})\Psi_Q$, corresponding to the spin-orbital interactions, which are not essential for the basic state of hadrons under consideration. The field Ψ_Q has standard non-relativistic normalization.

Further, the phenomenological experience in the potential quark models shows, that the kinetic energy of quarks practically does not depend on the quark contents of system, and it is determined by the color structure of state. So, we suppose that the kinetic energy is equal to $T = m_b v_b^2/2 + m_c v_c^2/2$ in the B_c-meson and to $T = m_d v_d^2/2 + m_l v_l^2/2$ in the case of doubly heavy baryons for the quark-diquark system, and it is $T/2 = m_b v_b^2/2 + m_c v_c^2/2$ in the diquark (the color factor of 1/2). Then

$$\frac{\langle B_c|\Psi_b^\dagger(i\boldsymbol{D})^2\Psi_b|B_c\rangle}{2M_{B_c}m_b^2} \simeq v_b^2 \simeq \frac{2m_c T}{m_b(m_c + m_b)} \tag{29}$$

$$\frac{\langle B_c|\Psi_c^\dagger(i\boldsymbol{D})^2\Psi_c|B_c\rangle}{2M_{B_c}m_c^2} \simeq v_c^2 \simeq \frac{2m_b T}{m_c(m_c + m_b)} \tag{30}$$

$$\frac{\langle \Xi_{QQ'}^\diamond|\Psi_Q^\dagger(i\boldsymbol{D})^2\Psi_Q|\Xi_{QQ'}^\diamond\rangle}{2M_{\Xi_{QQ'}^\diamond}m_Q^2} \simeq v_Q^2 \simeq \frac{2m_q T}{(m_q + m_{Q'} + m_Q)(m_{Q'} + m_Q)} + \frac{m_{Q'} T}{m_Q(m_Q + m_{Q'})}. \tag{31}$$

$$\frac{\langle \Xi^\diamond_{QQ'}|\Psi^\dagger_{Q'}(i\boldsymbol{D})^2\Psi_{Q'}|\Xi^\diamond_{QQ'}\rangle}{2M_{\Xi^\diamond_{QQ'}}m^2_{Q'}} \simeq v^2_{Q'} \simeq \frac{2m_q T}{(m_q+m_{Q'}+m_Q)(m_{Q'}+m_Q)} + \frac{m_Q T}{m_{Q'}(m_Q+m_{Q'})}. \tag{32}$$

Applying the quark-diquark approximation for the doubly heavy baryons and relating the matrix element of chromomagnetic interaction of heavy quarks in the B_c-meson and that of diquark with the light quark to the mass difference between the exited and ground states $M_{H^*} - M_H$, we have

$$\begin{aligned}
\frac{\langle B_c|\bar{c}c|B_c\rangle}{2M_{B_c}} &=& 1 - \frac{1}{2}v^2_c + \frac{3}{4}\frac{M_{B^*_c}-M_{B_c}}{m_c}\left(1-\frac{m_b}{2m_c}\right) + ... \\
&\approx& 1 - 0.190 + 0.037 - 0.061 + \ldots \tag{33} \\
\frac{\langle \Xi^\diamond_{cc}|\bar{c}c|\Xi^\diamond_{cc}\rangle}{2M_{\Xi^\diamond_{cc}}} &=& 1 - \frac{1}{2}v^2_c - \frac{1}{3}\frac{M_{\Xi^{\diamond *}_{cc}}-M_{\Xi^\diamond_{cc}}}{m_c} - \frac{5g^2}{18m^3_c}|\Psi^d(0)|^2 + ... \\
&\approx& 1 - 0.073 - 0.025 - 0.009 + \ldots \tag{34} \\
\frac{\langle \Omega^\diamond_{cc}|\bar{c}c|\Omega^\diamond_{cc}\rangle}{2M_{\Omega^\diamond_{cc}}} &=& 1 - \frac{1}{2}v^2_c - \frac{1}{3}\frac{M_{\Omega^{\diamond *}_{cc}}-M_{\Omega^\diamond_{cc}}}{m_c} - \frac{5g^2}{18m^3_c}|\Psi^d(0)|^2 + ... \\
&\approx& 1 - 0.078 - 0.025 - 0.009 + \ldots \tag{35}
\end{aligned}$$

Our presentation here is less detailed than in previous papers [12, 13]. However, we hope, that the interested reader can find there all needed details. Numerically, we have assigned $T \simeq 0.4$ GeV. The values of $|\Psi^d(0)|$ and M_H are given in the next section.

Analogous expressions can be obtained for the matrix elements of operator $Qg\sigma_{\mu\nu}G^{\mu\nu}Q$

$$\begin{aligned}
\frac{\langle B_c|\bar{c}g\sigma_{\mu\nu}G^{\mu\nu}c|B_c\rangle}{2M_{B_c}} &=& 3m_c(M_{B^*_c}-M_{B_c})\left(1-\frac{m_b}{2m_c}\right) \approx -0.216, \tag{36} \\
\frac{\langle \Xi^\diamond_{cc}|\bar{c}g\sigma_{\mu\nu}G^{\mu\nu}c|\Xi^\diamond_{cc}\rangle}{2M_{\Xi^\diamond_{cc}}m^2_c} &=& -\frac{4}{3}\frac{(M_{\Xi^{\diamond *}_{cc}}-M_{\Xi^\diamond_{cc}})}{m_c} - \frac{7g^2}{9m^3_c}|\Psi^d(0)|^2 \approx -0.124, \tag{37} \\
\langle \Omega_{QQ'}|\bar{c}g\sigma_{\mu\nu}G^{\mu\nu}c|\Omega_{QQ'}\rangle &=& \langle \Xi^\diamond_{QQ'}|\bar{c}g\sigma_{\mu\nu}G^{\mu\nu}c|\Xi^\diamond_{QQ'}\rangle \tag{38}
\end{aligned}$$

The permutations of quark masses lead to the required expressions for the operators of $\bar{b}b$ and $\bar{b}g\sigma_{\mu\nu}G^{\mu\nu}b$.

4 Numerical results and Discussion

The analysis of B_c-meson lifetime and lifetimes of doubly heavy baryons considered in [11, 12, 13] shows a strong dependence on quark mass values. As has been said in Introduction, the aim of this work is to reduce this uncertainty in a way for the case of doubly heavy baryons. For this purpose, we would like to note, that the OPE framework used by us for the calculation of lifetimes of doubly heavy baryons is a generalization of what was previously developed for theB_c-meson lifetime [11]. So we can test different sets of parameters used in the calculations of doubly heavy baryons lifetimes to the case of B_c-meson, for which we already have some experimental data.

The total width of B_c-meson consists of the spectator $\bar{b}$ and c-quark decays, Pauli interference and weak annihilation contributions. The lifetime dependence on the b-quark mass can be eliminated by the requirement, that for any value of m_c, the m_b mass value is obtained by the matching which

results in the B_d-meson lifetime to be equal to the experimentally measured value $\tau_{B_d} \approx 1.55$ ps. This prescription leads to the following approximate relation between the heavy quark masses

$$m_b = m_c + 3.5\text{GeV} \tag{39}$$

The performed analysis shows, that the B_c-meson lifetime dependence on the c-quark mass is quite large, and it is hard to eliminate the dependence by use of some external input. For example, a quite large c-quark mass or too low virtuality[3] are required to reproduce the absolute lifetime and semileptonic width in the OPE-based approach. Note also, that the application of OPE is based on the assumption about the quark-hadron duality. The latter can be violated in the case of D-mesons [29, 30], and, thus, in this case the OPE predictions are less reliable. On the other hand, as was noted by authors of [11], there is no obvious violation of this duality in the case of B_c-mesons. So, here we attempt to extract the c-quark mass value by fitting the OPE result for the B_c-meson lifetime to the experimentally measured value to use in the calculations of lifetimes for the doubly heavy baryons, yet to be discovered.

In the calculations of B_c-meson lifetime we have used the following set of parameters

$$m_s = 0.2\text{GeV} \quad |V_{cb}| = 0.04 \quad M_{B_c} = 6.26\text{GeV}$$
$$M_{B_c^*} - M_{B_c} = 0.073\text{GeV} \quad T = 0.37\text{GeV} \quad f_{B_c} = 0.5\text{GeV} \tag{40}$$

It is just the set of parameters, implemented in the original paper on the B_c lifetime. Here, we would like to note, that the quoted value of f_{B_c} is bigger, than one obtained in the framework of QCD sum rules [31, 32, 33].

In the calculation of quark spectator decays we put the renormalization scale to $\mu = m_Q$ and in the case of nonspectator decays we have [4] $\mu = m_{red}$, being the reduced mass for the B_c-meson system. In Table 1 we have collected the numerical values of B_c-meson lifetimes and relative spectator and nonspectator contributions for the different sets of b and c-quark masses, satisfying relation (39). Analyzing this Table, we see that, for example, the set of parameters, proposed in [14]

Parameters, GeV	$\sum b \to \bar{c}$, ps^{-1}	$\sum c \to s$, ps^{-1}	PI, ps^{-1}	WA, ps^{-1}	τ_{B_c}, ps
$m_b = 5.0, m_c = 1.5, m_s = 0.20$	0.694	1.148	-0.115	0.193	0.54
$m_b = 4.8, m_c = 1.35, m_s = 0.15$	0.576	0.725	-0.132	0.168	0.75
$m_b = 5.1, m_c = 1.6, m_s = 0.45$	0.635	1.033	-0.101	0.210	0.55
$m_b = 5.1, m_c = 1.6, m_s = 0.20$	0.626	1.605	-0.101	0.210	0.43
$m_b = 5.05, m_c = 1.55, m_s = 0.20$	0.623	1.323	-0.107	0.201	0.48
$m_b = 5.0, m_c = 1.5, m_s = 0.15$	0.620	1.204	-0.114	0.193	0.53

Table 1: The value of B_c-meson lifetime together with the spectator and nonspectator contributions to the width at various choices of parameters.

for the calculation of lifetimes for the doubly charmed baryons, is completely inconsistent with the

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[3]At low values, the perturbation theory cannot be justified.

[4]As in the case of B_c-meson, the nonspectator contributions are not so large as compared to those of doubly charmed baryons, we have not introduced an additional low energy logarifmic renormalization for them.

experimental data, when applied to the calculation of B_c-lifetime. At this set, we have $\tau_{B_c} = 0.75$ ps contrary to

$$\tau_{B_c}^{exp} = 0.46 \pm 0.18^{stat} \pm 0.03^{syst} \text{ps}. \tag{41}$$

The best set of parameters turns out to be $m_b = 5.05$, $m_c = 1.55$, $m_s = 0.2$ GeV. So, now we in the situation, when our set of parameters has a strong motivation, and we may, at a confidence level, apply it to the calculation of lifetimes for the doubly heavy baryons.

Estimating the lifetimes of doubly charmed baryons we put

$$
\begin{aligned}
& m_s = 0.2 \text{ GeV} \quad m_s^* = 0.45 \text{ GeV} \quad m_l^* = 0.3 \text{ GeV} \quad |V_{cs}| = 0.9745 \\
& M_{\Xi_{cc}^{++}} = M_{\Xi_{cc}^+} = 3.478 \text{ GeV} \quad M_{\Omega_{cc}} = 3.578 \text{ GeV} \\
& M_{\Xi_{cc}^{\diamond*}} - M_{\Xi_{cc}^\diamond} = M_{\Omega_{cc}^*} - M_{\Omega_{cc}} = 0.132 \text{ GeV} \quad T = 0.4 \text{ GeV} \quad \Psi^d(0) = 0.150 \text{ GeV}^{\frac{3}{2}}
\end{aligned}
\tag{42}
$$

The numerical values of parameters, characterizing the doubly charmed baryons, are taken from [15, 16]. m_s^* and m_l^* are the strange and light quark constituent masses, used for the calculation of bound state effects in the hadronic matrix elements of doubly charmed baryons. For the value of light quark-diquark function we assume

$$|\Psi^{dl}(0)|^2 = \frac{2}{3} \frac{f_D^2 M_D k^{-\frac{4}{9}}}{12}, \tag{43}$$

where $f_D = 170$ MeV. This expression was obtained by performing the steps similar to [34, 35] for the derivation of hyperfine splitting in the light quark-diquark system. The factor $k^{-\frac{4}{9}}$ accounts for the low energy logarithmic renormalization of f_D constant. In the calculation of nonspectator effects, we have accounted for the low energy logarithmic renormalization to a hadronic scale μ, where the latter was determined from the fit of theoretical predictions for the lifetime differences of baryons Λ_c, Ξ_c^+ and Ξ_c^0 to the corresponding experimental values. Tables 2,3,4 contain the values of lifetimes and relative spectator and nonspectator contributions for the doubly charmed baryons, as calculated at different sets of parameters used previously[12, 14], and that of obtained from the fit of B_c-meson lifetime to the experimentally measured value.

Parameters, GeV	$\sum c \to s$, ps^{-1}	PI, ps^{-1}	$\tau_{\Xi_{cc}^{++}}$, ps
$m_c = 1.35, m_s = 0.15$	1.638	-0.616	0.99
$m_c = 1.6, m_s = 0.45$	2.397	-0.560	0.56
$m_c = 1.55, m_s = 0.2$	3.104	-0.874	0.45

Table 2: The value of Ξ_{cc}^{++} lifetime together with the spectator and nonspectator contributions at various values of parameters.

The lifetimes, calculated at $m_c = 1.6$ GeV, $m_s = 0.45$ GeV, differ from those calculated previously at these values of parameters, because of different value for the wavefunction of light quark-diquark system, used previously. The lifetime of Ξ_{cc}^+ differs from that of calculated in [14] because of the weak scattering contribution to the total lifetime of baryon under consideration, which was wrongly estimated before in [14]. The final comment concerns with the importance of Pauli interference in semileptonic inclusive decays of c-quark, which was introduced by Voloshin. As argued

Parameters, GeV	$\sum c \to s$, ps^{-1}	WS, ps^{-1}	$\tau_{\Xi_{cc}^+}$, ps
$m_c = 1.35, m_s = 0.15$	1.638	1.297	0.34
$m_c = 1.6, m_s = 0.45$	2.397	2.563	0.20
$m_c = 1.55, m_s = 0.2$	3.104	1.776	0.20

Table 3: The value of Ξ_{cc}^+ lifetime together with the spectator and nonspectator contributions at various values of parameters.

Parameters, GeV	$\sum c \to s$, ps^{-1}	PI, ps^{-1}	$\tau_{\Omega_{cc}}$, ps
$m_c = 1.35, m_s = 0.15$	1.638	1.780	0.30
$m_c = 1.6, m_s = 0.45$	2.397	0.506	0.34
$m_c = 1.55, m_s = 0.2$	3.104	1.077	0.24

Table 4: The value of Ω_{cc}^+ lifetime together with the spectator and nonspectator contributions at various values of parameters.

by the authors of [14], this term can be valuable. So, it is given by the following equation:

$$\Gamma_{SL}^{Voloshin} = \frac{G_F^2}{12\pi}|V_{cd}|^2 m_c^2 (4\sqrt{k} - 1)5|\Psi^{dl}(0)|^2. \tag{44}$$

This term being doubly Cabbibo suppressed in the case of Ξ_{cc}^+-baryons, does not give any sizeable contribution, when we discuss the lifetimes of these baryons and should be taken into account only in estimations of semileptonic branching ratios of heavy baryons. In the case of Ω_{cc}-baryon it is no longer suppressed, and, so, this term is explicitly accounted for in our formulae.

As can be seen from the previously performed analysis [12, 13, 14], the lifetimes of doubly heavy baryons strongly depend on the value of wavefunction for the light quark-diquark system at the origin. Also, we can present some arguments for its determination, but in all the cases these arguments rest on the hypothesis of quark-diquark picture for the doubly heavy baryons. In Fig. 1, 2, 3 we have plotted the dependence of lifetimes for the doubly charmed baryons on this parameter. The precise value of light quark-diquark wavefunction is under question, and, thus, the uncertainty in its value should be included in the presented estimates of lifetimes.

Finally we would like to comment on the theoretical errors in the given results. They are mainly caused by the following:

1) The uncertainty in c-quark mass, taking into account the experimental errors on the B_c-meson lifetime, can lead to $\frac{\delta\Gamma}{\Gamma} \approx 15\%$.

2) The uncertainty in the value of light quark-diquark wave function at the origin can lead to $\frac{\delta\Gamma}{\Gamma} \approx 30\%$.

Combining these two sources of uncertainties we get the total error of presented estimates at the level of 45%.
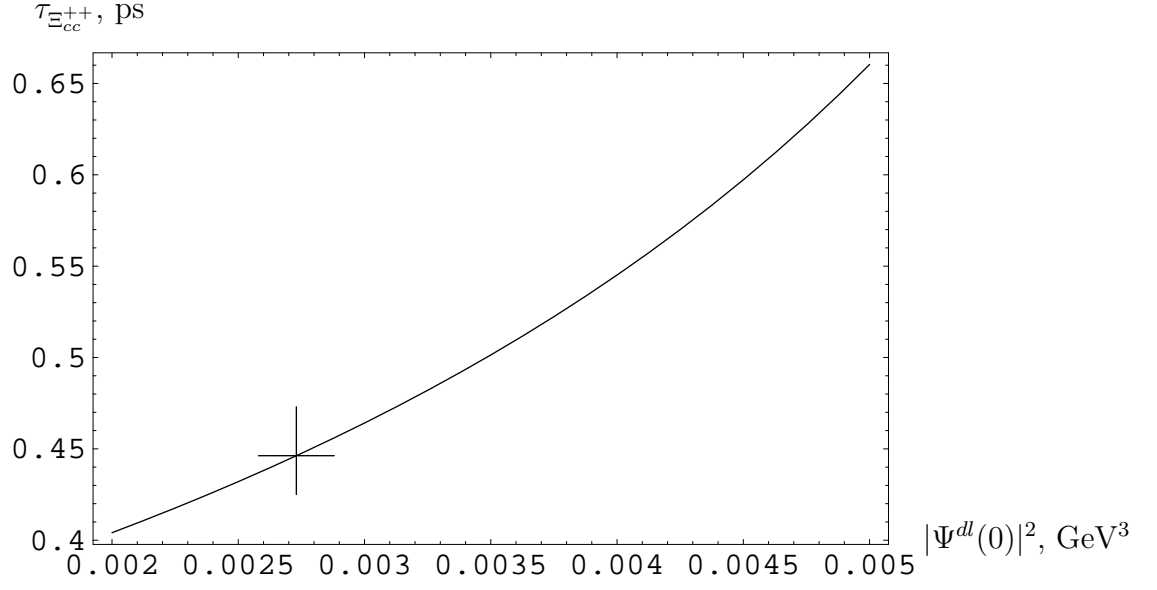
Figure 1: The dependence of Ξ_{cc}^{++}-baryon lifetime on the value of wavefunction of light quark-diquark system at the origin $|\Psi^{dl}(0)|$.
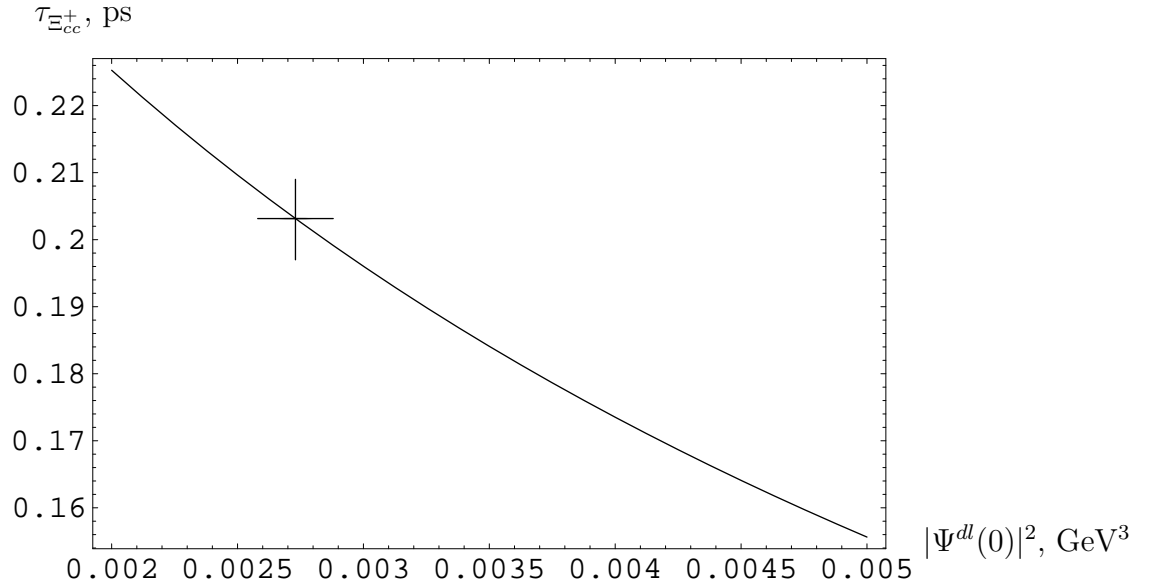

Figure 2: The dependence of Ξ_{cc}^{+}-baryon lifetime on the value of wavefunction of light quark-diquark system at the origin $|\Psi^{dl}(0)|$.
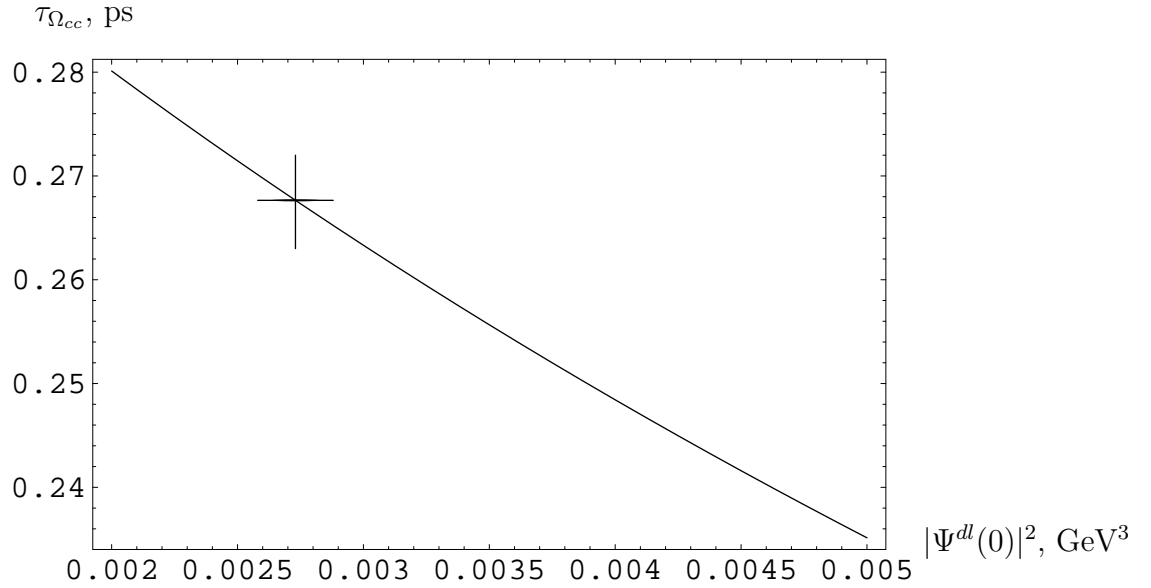
Figure 3: The dependence of Ω_{cc}-baryon lifetime on the value of wavefunction of light quark-diquark system at the origin $|\Psi^{dl}(0)|$.

5 Conclusion

In the present paper we have performed a detail investigation of parameter influence on the lifetimes of systems with two heavy flavors, calculated in the OPE-based approach. The fit to currently available data on the lifetimes of B_d and B_c-mesons, allowed us significantly to constraint the region of heavy quark masses. We present the numerical estimates for the lifetimes of doubly charmed baryons and discuss the huge difference between the results obtained previously.

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